New Steady State Kalman Filter for Tracking High Maneuvering Targets

Seyyed-Amin Hosseini[1]**, Mohammad Farrokhi**[1, 2]
[1] Department of Electrical Engineering
[2] Center of Excellence for Power System Automation and Operation
Iran University of Science and Technology, Tehran 16846-13114, IRAN
Phone: (+9821) 77240492, Fax: (+9821) 77240490
amin_hosseini@iust.ac.ir, farrokhi@iust.ac.ir

Abstract: This paper presents a new steady state Kaman filter for tracking high maneuvering targets. The $a-b$ and the $a-b-g$ filters are the steady-state Kaman filters for tracking constant speed and constant acceleration targets, respectively. However, these filters can not predict high maneuvering targets with good accuracy. The proposed filter is called $a-b-g-h$ filter. In this filter h is the gain of jerk for tracking of high maneuvering targets. Simulation results show good performance of the proposed filter as compared to $a-b-g$ filter to track the jerky movements.

Keywords: Target tracking, Steady-state estimation filters, $a-b$ filter, $a-b-g$ filter

Nomenclatures:

$\mathbf{x}$: state vector ($n\times 1$)
$\mathbf{F}$: transformation matrix ($n\times n$)
$\mathbf{u}$: process noise
v : measurement noise
x : position state
$\dot{x}$: velocity state
$\ddot{x}$: acceleration state
$\dddot{x}$: jerk state
T : sampling time
$\mathbf{h}$: observablility matrix ($3\times n$)
a : correlation parameter
Q_j : variance of process noise
$\mathbf{Q}$: covariance matrix of process noise ($n\times n$)
N : variance of target jerk
s_j^2 : variance of target jerk
R : variance of measurement noise
s_v^2 : variance of measurement noise
$\mathbf{P}$: state covariance matrix ($n\times n$)
P_{ij} : elements of $\mathbf{P}$
$\mathbf{S}$: updated state covariance matrix ($n\times n$)
$\mathbf{w}$: gain of filter
g_i : elements of $\mathbf{w}$
a : position gain
b : velocity gain
g : acceleration gain
h : jerk gain
l : tracking index

1. Introduction

The key point to successful target tracking is to extract useful information about the target state from the observed data. In order to achieve this goal, one needs a useful target model. The simplest model for a target, the so-called *white-noisy acceleration model* is used when the maneuver is small or random [1], [2]. The other simple model is the *Wiener-process acceleration model* that is referred to as the *constant-acceleration model* [1], [2]. On the other hand, the Singer acceleration model is a standard model for targets with maneuvers [3]. This model assumes that the target acceleration is a zero-mean stationary first-order Markov process. For high maneuvering targets, a jerky model is proposed by Mehrotra and Mahapatra [4]. In this method, the jerk is modeled as a zero-mean first-order Markov process, in the same way as the Singer acceleration model.

One of the widely used approaches for state estimation is the Kalman filter [5], [6]. However, Kalman filter imposes large amount of computations. In order to reduce computational burdens, constant-gains Kalman filters are used. The $a-b$ and the $a-b-g$ filters are the steady-state Kaman filters for target tracking with constant speeds and constant accelerations, respectively [7]-[10]. For high maneuvering targets (i.e. targets with changing accelerations), the performance of these steady-state filters deteriorates. To improve this shortcoming, an extension of these filters is proposed in this paper. This filter is called the $a-b-g-h$ filter, in which h is the gain for jerk. Simulation results show good performance of the proposed filter as compared to the $a-b-g$ filter for tracking jerky movements.

The rest of this paper is organized as follow. Section 2 defines the jerk model. Section 3 presents gain computations for the proposed filter. Section 4 gives the simulation results. Finally, Section 5 concludes the paper.

2. Jerk Model for Target Motion

There are many types of target motion, especially those involving the modern generation of highly maneuvering aerospace vehicles that call for better tracking performance than what is provided by acceleration models. The reason for the inadequate tracking performance of current models is that the higher order derivatives in the case of very highly maneuvering targets are not insignificant, leading to model inaccuracies when terms only up to the second or third orders derivatives are considered.

The state model for the target motion is defined by the following vector-matrix equations [4]:

$$\mathbf{x}(k+1) = \mathbf{F}(k)\mathbf{x}(k) + \mathbf{u}(k), \tag{1}$$

$$y(k+1) = \mathbf{h}(k+1)\mathbf{x}(k+1) + v(k+1), \tag{2}$$

where $\mathbf{x} = [x \quad \dot{x} \quad \ddot{x} \quad \dddot{x} \quad y \quad \dot{y} \quad \ddot{y} \quad \dddot{y} \quad z \quad \dot{z} \quad \ddot{z} \quad \dddot{z}]^T$ is the state vector, y is the output, $\mathbf{u}$ and v are white noises denoting the process and the measurement noises, respectively, all with appropriate dimensions. Moreover, the transition matrix and the observation vector are

$$\mathbf{F} = \left[\begin{array}{cccc|cccc|cccc}
1 & T & \frac{T^2}{2} & \frac{T^3}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & T & \frac{T^2}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 1 & T & \frac{T^2}{2} & \frac{T^3}{6} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & T & \frac{T^2}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & T & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & T & \frac{T^2}{2} & \frac{T^3}{6} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & T & \frac{T^2}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & T \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right] \tag{4}$$

$$\mathbf{h} = \left[\begin{array}{cccc|cccc|cccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0
\end{array}\right], \tag{5}$$

in which T is the sampling time interval. Matrix **h** shows that only the position sensor is used for target tracking. The covariance matrix for this system is equal to

$$\mathbf{Q}=Q_j\left[\begin{array}{cccc:cccc:cccc}
\frac{T^7}{252} & \frac{T^6}{72} & \frac{T^5}{30} & \frac{T^4}{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{T^6}{72} & \frac{T^5}{30} & \frac{T^4}{24} & \frac{T^3}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{T^5}{30} & \frac{T^4}{24} & \frac{T^3}{6} & \frac{T^2}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{T^4}{24} & \frac{T^3}{6} & \frac{T^2}{2} & T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hdashline
0 & 0 & 0 & 0 & \frac{T^7}{252} & \frac{T^6}{72} & \frac{T^5}{30} & \frac{T^4}{24} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{T^6}{72} & \frac{T^5}{30} & \frac{T^4}{24} & \frac{T^3}{6} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{T^5}{30} & \frac{T^4}{24} & \frac{T^3}{6} & \frac{T^2}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{T^4}{24} & \frac{T^3}{6} & \frac{T^2}{2} & T & 0 & 0 & 0 & 0 \\
\hdashline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{T^7}{252} & \frac{T^6}{72} & \frac{T^5}{30} & \frac{T^4}{24} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{T^6}{72} & \frac{T^5}{30} & \frac{T^4}{24} & \frac{T^3}{6} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{T^5}{30} & \frac{T^4}{24} & \frac{T^3}{6} & \frac{T^2}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{T^4}{24} & \frac{T^3}{6} & \frac{T^2}{2} & T
\end{array}\right], \qquad (6)$$

where $Q_j = 2as_j^2$ is the variance of the process noise in the jerk model, in which a is the correlation parameter and s_j^2 is the variance of the target jerk.

In the next section, for the ease of presentation, only the state variables along the x axis (i.e. x, $\dot{x}$, $\ddot{x}$, and $\dddot{x}$) will be considered. For other axes (i.e. y and z) the same equations apply. This fact can also be observed from matrices **F** and **Q** in (4) and (6), respectively.

3. The $a-b-g-h$ Filter

The following equations hold for the Kalman filter in steady-state conditions [9]:

$$\mathbf{P}(k\,|\,k)=\mathbf{P}(k-1\,|\,k-1),$$

$$\mathbf{P}(k+1\,|\,k)=\mathbf{P}(k\,|\,k-1),$$

$$\mathbf{w}(k)=\mathbf{w}(k-1),$$

where $\mathbf{w}$ is the gain of the filter. The components of the steady-state covariance matrix is denoted as

$$\lim_{k\to\infty} \mathbf{P}(k \mid k) = \left[\mathbf{P}_{ij}\right]. \tag{7}$$

Components of the one-step prediction of the covariance matrix are defined as

$$\lim_{k\to\infty} \mathbf{P}(k+1 \mid k) = \left[\mathbf{M}_{ij}\right]. \tag{8}$$

The updated covariance in Kalman filter is defined as

$$S = \mathbf{h}\mathbf{P}(k+1 \mid k)\mathbf{h}^T + R\,, \tag{9}$$

where $\mathbf{h}$ is the same as before, $R = s_v^2$, in which s_v^2 is the variance of the measurement noise. Using (5), (8) and (9), the updated covariance will be

$$S = m_{11} + R. \tag{10}$$

Hence, the Kalman filter gain becomes

$$\begin{aligned} \mathbf{w} &= \mathbf{P}(k+1 \mid k)\mathbf{h}^T S^{-1} \\ &= \begin{bmatrix} \dfrac{m_{11}}{m_{11}+R} & \dfrac{m_{12}}{m_{11}+R} & \dfrac{m_{13}}{m_{11}+R} & \dfrac{m_{14}}{m_{11}+R} \end{bmatrix}^T \\ &= \begin{bmatrix} g_1 & g_2 & g_3 & g_4 \end{bmatrix}^T. \end{aligned} \tag{11}$$

Using (11), it yields

$$m_{1i} = \frac{g_i}{1-g_1} R \qquad i = 1,2,3,4. \tag{12}$$

Hence, the updated covariance matrix becomes

$$\mathbf{P}(k+1 \mid k+1) = (\mathbf{I} - \mathbf{w}\mathbf{h})\mathbf{P}(k+1 \mid k). \tag{13}$$

Using (7), (8) and (13), it gives

$$[\mathbf{P}_{ij}] = (\mathbf{I} - \mathbf{w}\mathbf{h})[\mathbf{M}_{ij}]. \tag{14}$$

Therefore, $[\mathbf{P}_{ij}]$ is

$$[\mathbf{P}_{ij}] = \begin{bmatrix} (1-g_1)m_{11} & (1-g_1)m_{12} & (1-g_1)m_{13} & (1-g_1)m_{14} \\ (1-g_1)m_{12} & m_{22} - g_2 m_{12} & m_{23} - g_2 m_{13} & m_{24} - g_2 m_{14} \\ (1-g_1)m_{13} & m_{23} - g_2 m_{13} & m_{33} - g_3 m_{13} & m_{34} - g_3 m_{14} \\ (1-g_1)m_{14} & m_{24} - g_2 m_{14} & m_{34} - g_3 m_{14} & m_{44} - g_3 m_{14} \end{bmatrix}. \tag{15}$$

Hence, the covariance prediction equation in Kalman filter is

$$\mathbf{P}(k\,|\,k) = \mathbf{F}^{-1}\left[\mathbf{P}(k+1\,|\,k) - \mathbf{Q}\right](\mathbf{F}^{-1})^{T} = [\mathbf{P}_{ij}]. \tag{16}$$

If $[\mathbf{P}_{ij}] = \mathbf{P}$, then, according to (14) the elements of $\mathbf{P}$ are

$$\begin{aligned} P(1,1) = m_{11} - \frac{1}{126} a s_{j}^{2} T^{7} - 2Tm_{12} + T^{2}m_{13} - \frac{1}{3}T^{3}m_{14} + T^{2}m_{22} \\ -T^{3}m_{23} + \frac{1}{3}T^{4}m_{24} + \frac{1}{4}T^{4}m_{33} - \frac{1}{6}T^{5}m_{34} + \frac{1}{36}T^{6}m_{44}, \end{aligned} \tag{17}$$

$$\begin{aligned} P(1,2) = P(2,1) = m_{12} + \frac{1}{36}aNT^{6} - Tm_{22} + \frac{3}{2}T^{2}m_{23} - \frac{2}{3}T^{3}m_{24} \\ -Tm_{13} - \frac{1}{2}T^{3}m_{33} + \frac{5}{12}T^{4}m_{34} + \frac{1}{2}T^{2}m_{14} - \frac{1}{12}T^{5}m_{44}, \end{aligned} \tag{18}$$

$$\begin{aligned} P(1,3) = P(3,1) = m_{13} - \frac{1}{15}aNT^{5} - Tm_{23} + \frac{1}{2}T^{2}m_{33} \\ -\frac{2}{3}T^{3}m_{34} - Tm_{14} + T^{2}m_{24} + \frac{1}{6}T^{4}m_{44}, \end{aligned} \tag{19}$$

$$P(1,4) = P(4,1) = m_{14} + \frac{1}{12}aNT^{4} - Tm_{33} + \frac{1}{2}T^{2}m_{34} - \frac{1}{6}T^{3}m_{44}, \tag{20}$$

$$P(2,2) = m_{22} - \frac{1}{10}aNT^{5} - 2Tm_{23} + T^{2}m_{24} + T^{2}m_{33} - T^{3}m_{34} + \frac{1}{4}m_{44}, \tag{21}$$

$$P(4,4) = m_{44} - 2aNT, \tag{22}$$

$$P(2,3) = P(3,2) = m_{23} + \frac{1}{4}aNT^{4} - Tm_{33} + \frac{3}{2}T^{2}m_{34} - Tm_{24} + \frac{1}{2}T^{3}m_{44}, \tag{23}$$

$$P(3,3) = m_{33} - \frac{2}{3}aNT^{3} - 2Tm_{34} + T^{2}m_{44}, \tag{24}$$

$$P(3,4) = m_{34} + aNT^{2} - Tm_{44}. \tag{25}$$

where $N = s_{j}^{2}$ and s_{j}^{2} was defined before.

On the other hand, according to (15), elements of $[\mathbf{P}_{ij}]$, which are designated with $PP(i,j), (i = j = 1, \mathbf{K}, 4)$ for convenience, are equal to

$$PP(1,l) = PP(l,1) = m_{1l} - m_{1l}g_{1}, \quad l = 1,2,3,4, \tag{26}$$

$$PP(2,j) = PP(j,2) = m_{2j} - m_{2j}g_{2}, \quad j = 2,3,4, \tag{27}$$

$$PP(1,k) = PP(k,1) = m_{3k} - m_{1k}g_3, \quad k = 3,4, \tag{28}$$

$$PP(4,4) = m_{44} - m_{14}g_4. \tag{29}$$

Equating $P(i,j)$ and $PP(i,j)$ yields

$$m_{44} = \frac{1}{T}\frac{g_3 g_4}{1-g_1}R + aNT, \tag{30}$$

$$m_{34} = \frac{1}{2T}\frac{g_3^2}{1-g_1}R + \frac{1}{6}aNT^2 + \frac{1}{2}\frac{g_4 g_3}{1-g_1}R, \tag{31}$$

$$m_{34} = \frac{1}{T}\frac{g_2 g_4}{1-g_1}R + \frac{1}{6}aNT^2 + \frac{1}{2}\frac{g_4 g_3}{1-g_1}R, \tag{32}$$

$$m_{24} = -\frac{1}{12T}R\frac{12g_1g_4 + 3Tg_3^2 + T^2g_3g_4}{(g_1-1)}, \tag{33}$$

$$m_{33} = -\frac{1}{6T}R\frac{6g_3g_2 + 3Tg_3^2 + T^2g_3g_4 - 6g_1g_4}{(g_1-1)}, \tag{34}$$

$$\begin{aligned} m_{23} = -\frac{1}{90T}(&90Rg_1g_3 - aNT^5 + aNT^5g_1 + 45TRg_2g_3 \\ &+15T^2Rg_3^2 + 45TRg_1g_4 - 90TRg_4)/(g_1-1), \end{aligned} \tag{35}$$

$$\begin{aligned} m_{22} = -\frac{1}{360T}(&-360TRg_3 - aNT^6 - 360T^2Rg_4 + 360g_1g_2R + 15T^3g_3^2R \\ &+90T^2Rg_2g_3 + 210T^2Rg_1g_2 + 540TRg_1g_3 + aNT^6g_1 - 5T^4Rg_3g_4)/(g_1-1). \end{aligned} \tag{36}$$

It should be noted that two different m_{34} are obtained in (31) and (32), which is due to the different elements of $P(i,j)$ and $PP(i,j)$. Equating (17) and (26) for l=1 and using (30) to (36) gives

$$\begin{aligned} &-aNT^7 + aNT^7g_1 - 1260g_1g_3T^2 - 420T^3Rg_1g_4 - 2520g_1g_2R \\ &+5040Tg_2R + 840T^3g_4R - 2520g_1^2R = 0. \end{aligned} \tag{37}$$

The gains of this filter must cancel out the effects of T in (37). The gains are selected as [1]

$$\mathbf{w} = \begin{bmatrix} a & \frac{b}{T} & \frac{g}{2T} \end{bmatrix} = [g_1 \quad g_2 \quad g_3]. \tag{38}$$

Then, using (31) and (32) g_4 can be calculated as

$$g_4 = \frac{g_3^2}{2g_2}. \qquad (39)$$

Therefore, according to (38) and (39), g_4 is equal to

$$g_4 = \frac{h}{8T^3}, \qquad (40)$$

where

$$h = \frac{g^2}{b}. \qquad (41)$$

In steady-state tracking filters, gains are computed based on the tracking index. To compute the tracking index and the gains, some additional equations are needed. By equating (22) and (29), another form of m_{14} can be obtained as

$$m_{14} = \frac{2aNT}{g_4}. \qquad (42)$$

Using (12) and (42), a new set of equations for m_{ij} are obtained as follows:

$$m_{44} = \frac{2aN}{g_4} g_3 + aNT \qquad (43)$$

$$m_{34} = \frac{1}{2T} \frac{Rg_3^2}{1-g_1} + \frac{1}{6} aNT^2 + aNT \frac{g_3}{g_4} \qquad (44)$$

$$m_{34} = 2aN \frac{g_2}{g_4} + \frac{1}{6} aNT^2 + aNT \frac{g_3}{g_4} \qquad (45)$$

$$m_{24} = 2aN \frac{g_1}{g_4} + \frac{1}{4} \frac{g_3^2}{1-g_1} + \frac{1}{6} aNT^2 \frac{g_3}{g_4} \qquad (46)$$

$$m_{33} = \frac{1}{T} \frac{Rg_3 g_2}{1-g_1} + \frac{1}{2} \frac{Rg_3^2}{1-g_1} + \frac{1}{3} aNT^2 \frac{g_3}{g_4} - 2aN \frac{g_1}{g_4} \qquad (47)$$

$$m_{23} = \frac{1}{2T} R \frac{g_2^2}{1-g_1} - \frac{1}{120} aNT^4 + \frac{1}{8} TR \frac{g_3^2}{1-g_1} + \frac{1}{2} R \frac{g_2 g_3}{1-g_1} \qquad (48)$$

$$m_{22}=\frac{1}{T}\frac{Rg_1g_2}{1-g_1}+\frac{1}{720}aNT^5+\frac{3}{4}R\frac{g_2^2}{1-g_1}-\frac{1}{48}RT^2\frac{g_3^2}{1-g_1}$$
$$+\frac{1}{4}RT\frac{g_2g_3}{1-g_1}-\frac{1}{3}aNT^2\frac{g_1}{g_4}-\frac{1}{36}aNT^4\frac{g_3}{g_4} \tag{49}$$

Equating (17) and (26) for $l=1$ and using (42) to (49) gives

$$\frac{Tg_1g_2R}{1-g_1}-\frac{1}{6}\frac{T^4aNg_1}{g_4}+\frac{1}{3}\frac{T^4aN}{g_4}+\frac{aNT^7}{560}-\frac{2TRg_2}{1-g_1}$$
$$+\frac{1}{4}\frac{T^2Rg_2^2}{1-g_1}+\frac{1}{48}\frac{T^4Rg_3^2}{1-g_1}+\frac{Rg_1^2}{1-g_1}=0. \tag{50}$$

To determine the tracking index, equating (42) and (12) for $i=4$ results in

$$l^2=\frac{aNT^7}{R}=\frac{h^2}{128(1-a)}. \tag{51}$$

Using the obtained gains and the tracking index in (51) and replacing them into (37) and (50) yields

$$-\frac{1}{128}h^2-630ag-\frac{105}{2}ah-2520ab+5040b+105h-2520a^2=0 \tag{52}$$

$$\frac{1}{71680}h^2+\frac{1}{48}h+ab-\frac{1}{96}ha+a^2-2b+\frac{1}{4}b^2-\frac{1}{192}g^2=0 \tag{53}$$

Now, using (41), (51), (52) and (53), it gives

$$-16+16a+\frac{164}{105}l^4-\frac{1544}{315}l^4a+\frac{596}{105}l^4a^2+\frac{8}{3}l^2a^2-\frac{304}{105}l^4a^3+\frac{16}{105}l^6$$
$$-4l^2a^3-\frac{8}{15}l^6a^2+\frac{24}{35}l^6a^2-\frac{8}{21}l^6a^3-\frac{16}{1225}l^8a+\frac{24}{1225}l^8a^2-\frac{16}{1225}l^8a^3$$
$$+\frac{4}{1225}l^8a^4+\frac{8}{105}l^6a^4+\frac{179}{315}l^4a^4+\frac{4}{3}l^2a^4+\frac{4}{1225}l^8+a^4=0 \tag{54}$$

By calculating l (using $l^2=aNT^7/R$) and putting that into (54), a can be obtained. Then, h can be calculated using (51). Finally, b and g can be calculated using the following equations:

$$b=-(2a-4-\frac{1}{192}h)-\frac{1}{2}\Big[(4a-8-\frac{1}{48}h)^2$$
$$-4\left(\frac{1}{17920}h^2+\frac{1}{12}h-\frac{1}{24}ha+4a^2\right)\Big]^{1/2} \tag{55}$$

$$g = \sqrt{hb} \tag{56}$$

It should be noted that, in the process of finding these gains, the equations usually yield multiple answers, form which only the positive answers are acceptable. Moreover, from the predefined range of each gain, one can find the correct answer form the multiple positive solutions.

4. Simulation Results

In this section, a comparison between the proposed filter and the $a - b - g$ filter is demonstrated throughout simulations. The variance of the process noise for the jerk model is $Q_j = 2as_j^2$ where $s_j = 0.1m/s^3$. It is assumed that the radar measurement sequences are transformed from the polar coordinates to the Cartesian coordinates before the track-while-scan (TWS) process takes place [1]. The measurement covariance matrix is defined as

$$RR = \begin{bmatrix} R_{11} & 0 & 0 \\ 0 & R_{22} & 0 \\ 0 & 0 & R_{33} \end{bmatrix} \tag{57}$$

where R_{11}, R_{22} and R_{33} are variances in *x*, *y*, and *z* directions, respectively, and are equal to 2500 m^2. Moreover, the correlation parameter is $a = 0.7$ and the sampling time interval is *T* = 0.5 sec., which is the time of the radar antenna scanning a revolution. The jerk model, which is used for all filters, is the same model as Mehrotra and Mahapatra have proposed in [4]. For the initial position equal to $\begin{bmatrix} 0 & 0 & 1000 \end{bmatrix}^T$ m, and the initial velocity of $\begin{bmatrix} -1000 & 10 & 0 \end{bmatrix}^T$ m/sec, the gains are calculated as

$$a = 0.9493,\ b = 1.2004$$

$$g = 0.0213,\ h = 3.7695\text{e}-004$$

It should be noted that the gains of the $a - b - g$ filter are not necessarily the same as the proposed $a - b - g - h$ filter (although the derivation procedure is similar). The reader may refer to reference [1] on how to find the gains of the $a - b - g$ filter.

Table 1: Jerk of the target for different time intervals.

time interval		target movement
From	To	
$t = 0$ sec.	$t = 250$ sec.	Jerk = $0\ \text{m/s}^3$
$t = 251$ sec.	$t = 500$ sec.	Jerk = $0.25\ \text{m/s}^3$

Table2: RMSE in x, y and z axes in 500 sec

	RMSE in x axis	RMSE in y axis	RMSE in z axis
Proposed filter	50.18	42.56	43
$a-b-g$ filter	123.3675	141	202

The jerk of the target for different time intervals is summarized in Table 1. Since the most important issue in target tracking applications is the position error, only position errors are shown in simulation results.

Figs. 1–3 show the position errors in the *x*, *y*, and *z* directions, respectively. As these figures show, when the jerk is zero (i.e. the target is moving with constant acceleration) the $a-b-g$ filter performs slightly better than the proposed filter in this paper. On the other hand, when the target has jerky movements, the error of $a-b-g$ filter increases or even may loose the target. The reference tracking path for the $a-b-g-h$ filter is depicted in Fig. 6 in three dimensions. It should be mentioned that the same tracking path is used for the $a-b-g$ filter. For comparison, the Root-Mean-Square (RMS) errors for 500 sec. of simulations are given in Table 2.

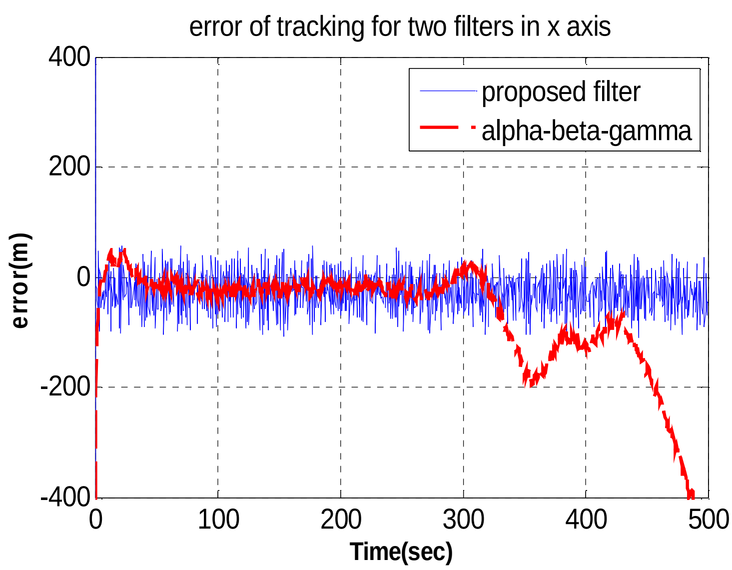

Fig. 1: Position tracking error on the x-axis

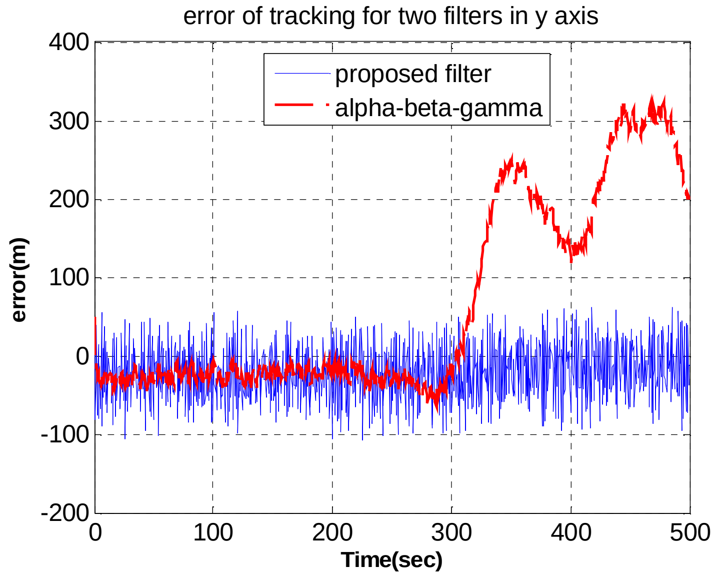

Fig. 2: Position tracking error on the y-axis

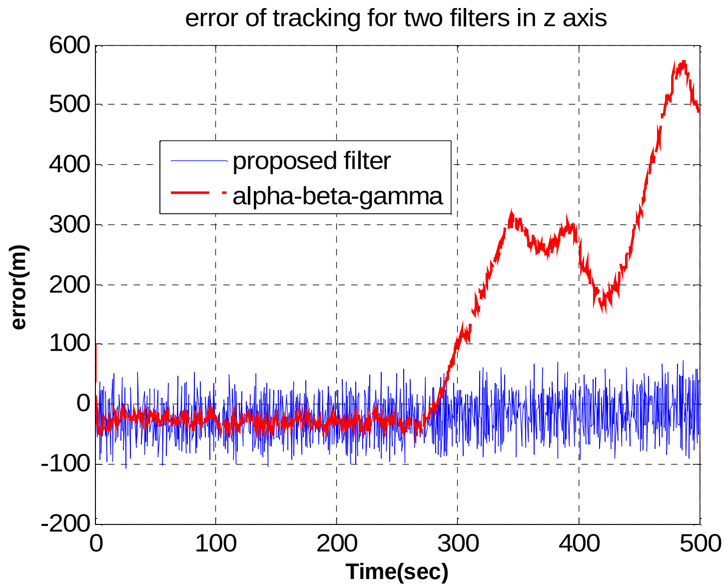

Fig. 3: Position tracking error on the z-axis

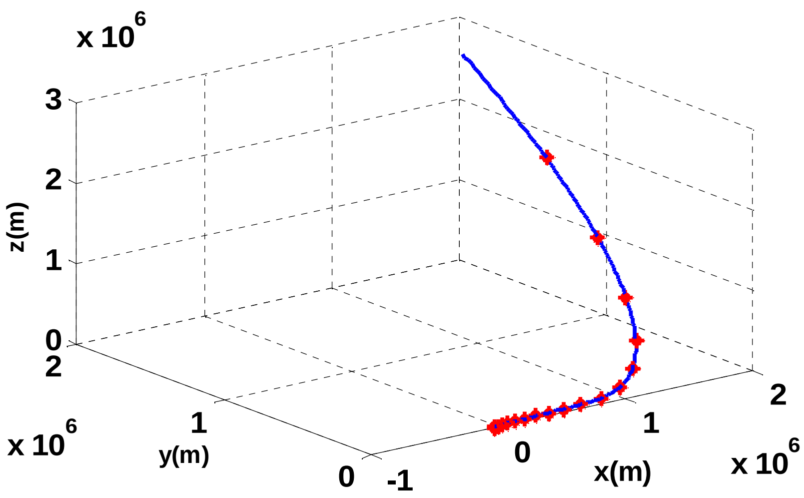

Fig.4: The model path and the tracking path in 3D

5. Conclusion

In this paper, a new filter, called the $a-b-g-h$ filter, was introduced. The proposed filter is an extension of the $a-b-g$ filter, where h is the gain for the target jerk. These filters are constant-gain filters and have lower calculation volume as compared to the Kalman filters. In addition, their tracking accuracy is acceptable. It was shown by simulations that the proposed filter can follow jerky models with high maneuvering properties, with good accuracy as compared to the $a-b-g$ filter.

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